Section 9.5 Common Logs and Natural Logs

Common Logs are used in chemistry and astronomy. They are written: $\log_{10} x$.

Common Logs use base 10.

Natural Logs are used in calculus and are written: $\log_e x = \ln x$. $e \approx 2.718$.

Your calculator has Log and Ln keys. Both of these logs are included on scientific calculators.

Thus far we have one log definition and 3 log properties:

$$\log_b a = x \Leftrightarrow b^x = a$$
$$\log_b (xy) = \log_b x + \log_b y$$
$$\log_b \frac{x}{y} = \log_b x - \log_b y$$
$$\log_b x^n = n \log_b x$$

The 4th and last log property is called the "base change" property.

$$\log_b a = \frac{\log_c a}{\log_c b}$$

Notice that the original base b is the argument for the log in the denominator and the original argument is the argument of the log in the numerator. Finally, notice that the new base, c, is the same both on top and on the bottom.

What would you get if instead of using base c you used base a?

$$\log_b a = \frac{\log_c a}{\log_c b}$$
$$\log_b a = \frac{\log_a a}{\log_a b}$$
$$\log_b a = \frac{1}{\log_a b}$$

The base change property is especially handy because calculators have ONLY common and natural logs. Suppose you need to evaluate $\log_4 5 = x$. This is, 4 to some power is 5. Find that power.

To do this on a calculator you need to do a base change thus:

$$\log_4 5 = \frac{\log_{10} 5}{\log_{10} 4} = \frac{.69897}{.60206} \approx 1.16$$

So this means that 4 raised to the 1.16 power will equal 5.

On your calculator, press 4. press power . put in 1.16 and press equals.

What if we had used natural logs instead of common logs?

$$\log_4 5 = \frac{\ln 5}{\ln 4} = \frac{.1.6094}{1.3863} \approx 1.16$$

Section 9.6 Solving Exponential Equations

$$\begin{cases} 4^{3x-5} = 16 \\ 4^{3x-5} = 4^2 \end{cases} \Rightarrow \quad \begin{aligned} 3x - 5 &= 2 \\ 3x &= 7 \\ x &= \frac{7}{3} \end{aligned}$$

$$\begin{cases} 5^x = 12 \\ \log\left(5^x\right) = \log(12) \\ x\log 5 = \log(12) \\ x = \dfrac{\log(12)}{\log 5} = 1.544 \end{cases}$$

Notice that this is a "division of logs" rather than a "log of a division".

$$\begin{cases} 2^{x^2+4x} = \dfrac{1}{8} \\ 2^{x^2+4x} = 2^{-3} \end{cases} \Rightarrow \quad \begin{aligned} x^2 + 4x &= -3 \\ x^2 + 4x + 3 &= 0 \\ (x+1)(x+3) &= 0 \\ x \ \varepsilon \ &\{-1,-3\} \end{aligned}$$

$$\begin{aligned} \log_4(8x-8) &= 3 \\ 8x - 8 &= 4^3 \\ 8x &= 64 + 8 \\ 8x &= 72 \\ x &= 9 \end{aligned}$$

$\log x + \log(x-3) = 1$

$\log\left(x(x-3)\right) = 1$ log without a base showing is a common log ie. $\log_{10}$

$x(x-3) = 10^1$ I put the exponent one here but normally we do not write a one exponent.

$$e^{0.06t} = 1500$$

$\ln\left(e^{0.06t}\right) = \ln(1500)$ remember $\log_e$ is ln (natural log)

$0.06t\ln(e) = \ln 1500$ We moved the exponent down to the coefficient position.

$0.06t(1) = \ln 1500$ $\ln e = \log_e e$ meaning e to some power is e. Thus 1.

$$t = \frac{\ln 1500}{0.06}$$

$$t = \frac{7.3132}{0.06}$$

$$t = 121.887$$

The text points out that the properties of logarithms are often needed. The goal is to first write an equivalent equation in which the variable appears in just one logarithmic expression.
We want this because we will use
the "definition of logarithms", $\log_b a = x \quad \Leftrightarrow \quad b^x = a$ to remove the logarithm from the equation.

$$\log_2(x+7) - \log_2(x-7) = 3$$

$$\log_2\left(\frac{x+7}{x-7}\right) = 3$$

$$2^3 = \frac{x+7}{x-7}$$

$$8 = \frac{x+7}{x-7}$$

$$8x - 56 = x + 7$$

$$7x = 63$$

$$x = 9$$

$$\log_7(x+1) + \log_7(x-1) = \log_7 8$$

$$\log_7\left(x^2 - 1\right) = \log_7 8$$

$$x^2 - 1 = 8$$

$$x^2 = 9$$

$$x = \pm 3$$

Why is $x = -3$ a "reject" solution?
Because the log must have a positive argument

another example:

$$\begin{cases} \log_{10} x + \log_{10}(x+3) = 1 \\ \log_{10} x(x+3) = 1 \\ x^2 - 3x = 10 \\ x^2 - 3x - 10 = 0 \\ (x-5)(x+2) = 0 \\ x = 5 \quad \text{or} \qquad x = -2 \\ \text{only} \qquad\quad \text{reject} \\ \text{solution} \qquad \text{domain restriction} \end{cases}$$

Section 10.1 Conic Sections: Parabolas, Circles

"Conic sections" are sections of conics – that is Two cones are placed together at the cusp of the cones as shown on page 650 of our text.

A parabola is the line formed when a plane intersects **one** of the cones. This plane is parallel to the edge of the cone so it only cuts one cone.

A circle is the line formed when a plane intersects one of the cones perpendicular to the axis of the cone.

If the plane is tipped but not parallel to side of the cone an ellipse is formed.

If the plan is parallel to the axis of the cone, a hyperbola is formed.

Don't be confused into thinking that a hyperbola is just two parabolas! The rate of the opening of the hyperbola parts is different than the rate of opening for a parabola.

In 10.1 we are concerned only with parabolas and circles.

In Chapter 8 we talked about the vertex of a parabola. The vertex was the highest or lowest point on the graph of the parabola. We found it to have coordinates (h,k) when we wrote the equation in "graphing form" $y = a(x-h)^2 + k$. Actually, when we completed the square in the equation: $y = ax^2 + bx + c$. We ended up with $y = a\left(x - \frac{-b}{2a}\right)^2 + c - \frac{b^2}{4a}$.

With this formula we know that the coordinates of the vertex will be $(h,k) = \left(\frac{-b}{2a}, c - \frac{b^2}{4a}\right)$.

These parabolas were functions because for each x value, there was a particular y value.

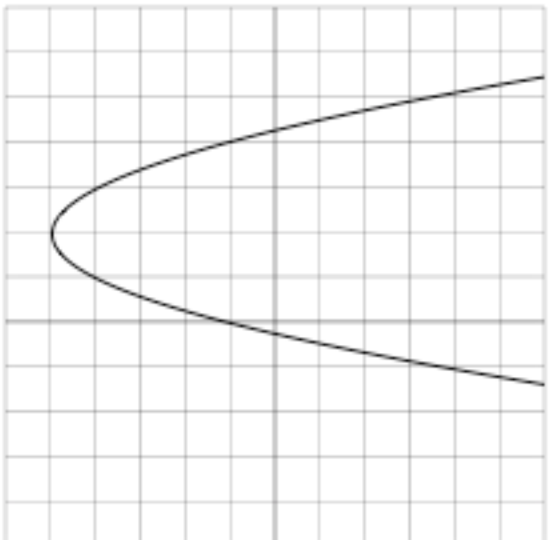
There are other kinds of parabolas.

The parabola graphed below is *not* a function and opens either left or right. We use $x = a(y-k)^2 + h$ as its graphing form.

For instance: $x = y^2 - 4y - 1$

$x = y^2 - 4y + 4 - 4 - 1$

$x = (y-2)^2 - 5$



We can see the vertex will be (– 5, 2).
The formula for finding the vertex without graphing for this type of parabola is remarkably similar to our earlier formula.

We need to complete the square for $x = ay^2 + by + c$

$x = a\left(y - \frac{-b}{2a}\right)^2 + c - \frac{b^2}{4a}$ so the coordinates for the vertex are: $(h,k) = \left(c - \frac{b^2}{4a}, \frac{-b}{2a}\right)$!

We would find the axis intercepts as you would expect:

$$y^2 - 4y - 1 \overset{set}{=} 0 \qquad \begin{array}{l} a = 1 \\ b = -4 \\ c = -1 \end{array}$$

Compute the axis intercepts: $x = \dfrac{4 \pm \sqrt{16+4}}{2}$

$$x = \frac{4 \pm 2\sqrt{5}}{2}$$

$$x = 2 \pm \sqrt{5}$$

How big is $\sqrt{5}$?

$$\sqrt{4} < \sqrt{5} < \sqrt{9}$$

$$2 < \sqrt{5} < 3$$

so $2 + \sqrt{5}$ is a little bigger than 4 and $2 - \sqrt{5}$ is negative and between –1 and zero.

Does our picture reflect this information?

The graphing form for a circle is: $(x-h)^2+(y-k)^2=r^2$ where the center of the circle has coordinates (h, k) and the radius of the circle is r.

To put an equation into graphing form often requires "completing the square" twice.

Given:

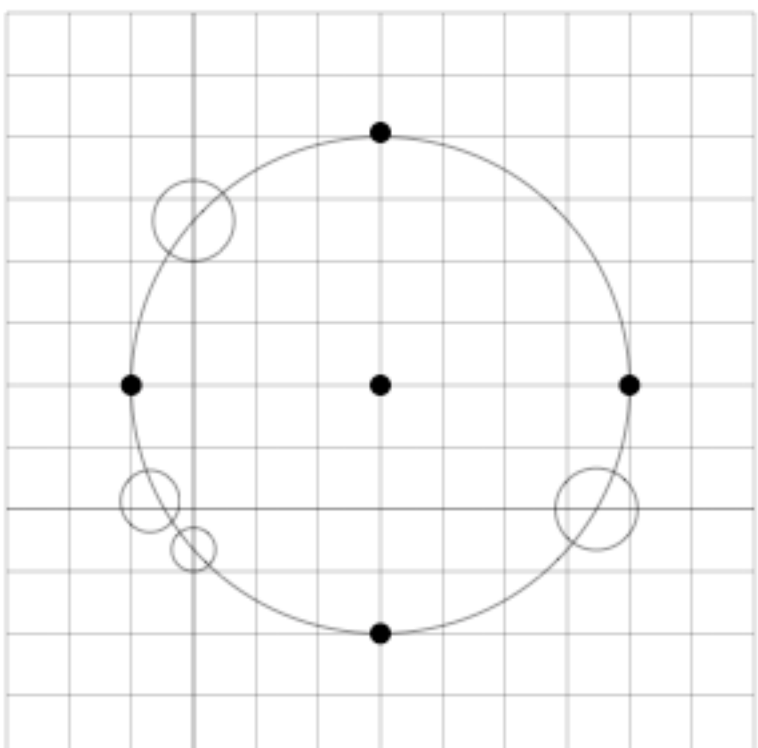
$x^2+y^2-6x-4y-3=0$ group x variables and group y variables

$x^2-6x+\quad +y^2-4y+\quad =3$ Complete the square twice

$x^2-6x+\ 9\ +y^2-4y+4=3+9+4$

$(x-3)^2+(y-2)^2=4^2$

We have a circle with center at (3, 2) and it has a radius of 4



I have circled the axis intercepts. How do we find those values? The x-intercept is when $y=0$ and the y-intercept is when $x=0$:

$(x-3)^2+(y-2)^2=4^2$	$(x-3)^2+(y-2)^2=4^2$
$(x-3)^2+(0-2)^2=4^2$	$(0-3)^2+(y-2)^2=4^2$
$(x-3)^2+4=16$	$9+(y-2)^2=16$
$(x-3)^2=12$	$(y-2)^2=7$
$x-3=\pm\sqrt{12}$	$y-2=\pm\sqrt{7}$
$x=3\pm 2\sqrt{3}$	$y=2\pm\sqrt{7}$

$$\sqrt{9}<\sqrt{12}<\sqrt{16}$$

Since $3<\sqrt{12}<4$

$$3<2\sqrt{3}<4$$

$$\sqrt{4}<\sqrt{7}<\sqrt{9}$$

$$2<\sqrt{7}<3$$

we know one the *x*- intercept is slightly more than 6 and the other is between negative one and zero. Likewise one *y*-intercept is between 4 and 5 and the other is between negative one and zero.

Circles are also defined as all those points that are the same distance from a single point called the "center."

The general distance formula is $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

If the center has coordinates (h,k) we get $\sqrt{(x-h)^2+(y-k)^2}=r$

This becomes $(x-h)^2+(y-k)^2=r^2$. Our graphing form for a circle!

Example

Given points $(-3,1)$ and $(5,21)$. These point are on opposite sides of a circle. Find the equation of that circle.

Need formula for center of a line: $\left(\frac{x_2+x_1}{2},\frac{y_2+y_1}{2}\right)$ yields $(1,11)$.

Need formula for radius (i.e. distance from center to one of the points)

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{(11-1)^2+(1+3)^2}$$

$$d=\sqrt{100+16}$$

$$d=\sqrt{116}$$

$$d^2=116$$

Therefore equation of circle is

$$(x-h)^2+(y-k)^2=d^2$$

$$(x-1)^2+(y-11)^2=116$$